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Bose Gas in Gravitational Field

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The Bose gas in an arbitrary curved space-time is considered. A method of construction of the thermodynamic potential of a quantum gas by means of a finite-temperature Green's function is proposed. On this basis the Bose and Boltzmann distributions are derived. The behavior of the chemical potential is investigated. The phenomenon of Bose–Einstein condensation is discussed.

1. INTRODUCTION

Different variants of the construction of nonstationary relativistic thermodynamics have been proposed (Israel and Kandrup, 1984; Bampi and Morro, 1980; Pavon *et al.*, 1982). The necessity of such a theory is connected with problems concerning the processes inside stars and the thermodynamics of early stages of the universe. Generalization of the standard thermodynamic and statistical equations for curved space-time and calculation of the possible dissipation processes were presented in the above works. According to the authors their methods can be applied to systems of classical particles. At the same time success in the construction of realistic cosmological models is generally connected with the calculation of quantum effects (Guth, 1981; Linde, 1984; Starobinsky, 1982). In this connection it is only natural to study some aspects of quantum statistics in curved space-time. In this article we show a method that enables us to construct statistics and to study some peculiarities of the thermodynamic behavior of a quantum Bose gas in external curved space-time. The method of finite-temperature quantum field theory can be used as a basic tool for this goal.

The close connection between quantum field theory and statistical mechanics has long been known. Thus in the works of Fradkin (1965), Fivel (1971), and Symanzik (1966) it was shown that the Euclidean

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Green's functions are similar to the distribution functions in statistical mechanics. The discovered analogies between statistical physics and quantum field theory at finite temperature were further strengthened after the thermodynamic equations for infinite equilibrium systems of free scalar and spinor fields were studied by Dolan and Jackiw (1974). These analogies provide an opportunity to construct statistical mechanics and thermodynamics for a material system in space-time with nontrivial topology and non-Euclidean geometry, since the field theory can be generalized for these cases. Such an approach was used by some authors for studying the finite-temperature effective potential in a space-time of constant curvature with boundaries (Denardo and Spallucci, 1983; Kennedy, 1981) and for a scalar field in a spherically symmetric model when the curvature depends on time only (Hu, 1982, 1983; Kennedy *et al.*, 1980).

At the same time the problem of the construction of the thermodynamics of infinite systems in an arbitrary curved space-time has not been solved properly so far. Probably the reasons for that are connected with the difficulties of defining temperature in such a way as is done in nonrelativistic equilibrium thermodynamics and the impossibility of defining the energy spectrum and vacuum state, as the energy levels depend on the geometry and topology of the space-time manifold. Therefore it is necessary to find another way for solving these problems. We believe that success can be achieved by using the Green's function method in the setting of thermodynamic laws. The knowledge of the Green's functions allows one to find the thermodynamical potential, occupation numbers, chemical potential, and other quantities quite easily. Besides, the equations for the Green's functions in curved space-time are well known (Birrell and Davies, 1982). The main task is to calculate the temperature dependence of the Green's functions of any material system in an external curved space-time.

Our assumption does not require getting thermodynamic laws and equations in the whole space-time manifold. It is better to divide the whole space-time volume occupied by the system into quasiequilibrium subsystems. One can consider conditions inside each subsystems to be in equilibrium, but one should take into account the fact that this state might be limited because of the external influence of the rest of the system. In this picture all the thermodynamic potential of the subsystem will depend on the point of space-time, but nevertheless it gives us an opportunity to study the change of the thermodynamic characteristics of an infinite system within the sphere of an observer.

The chief goal of this paper is to develop the quantum statistics and thermodynamics of a Bose gas in an arbitrary curved space-time in the adiabatic approximation by means of the finite-temperature Green's functions technique.

Our work is organized in the following way. In Section 2 we review a real scalar field in a curved space-time and give the basis of the construction of the local statistics and thermodynamics in term of Green's functions.

In Section 3 we describe the momentum space representation of the Feynman propagator of the scalar field.

Sections 4 and 5 are devoted to calculations of the finite-temperature Green's function, the thermodynamic potentials, occupation numbers for bosons, and the chemical potential.

The phenomenon of Bose–Einstein condensation in external curved space-time is considered in Section 6.

Section 7 is devoted to divergences in the one-loop effective action at finite temperature.

In the last section we give a short summary of our work.

2. BASIC FORMALISM

Let us consider a real scalar field φ with the Lagrangian

$$\mathscr{L} = -\tfrac{1}{2}\varphi(-\nabla_\mu\nabla^\mu + m^2 + \xi R)\,\varphi \tag{1}$$

where ∇_μ denotes the covariant derivative, R is the scalar curvature of the space-time, ξ is an arbitrary real number, and m is the mass. We use units with $\hbar = c = 1$ and metric signature $(-, +, +, +)$.

In the case of the noninteracting scalar field (1) the one-loop effective action is defined in the following way:

$$W = \frac{i}{2}\ln \operatorname{Det} K = -\frac{i}{2}\operatorname{tr}\ln G \tag{2}$$

where $KG = 1$, the operator K is given by

$$K_{xx'} = (-\nabla_\mu\nabla^\mu + m^2 + \xi R)\,\delta(x - x') \tag{3}$$

and the Green's function $G(x, x')$ obeys the equation

$$(-\nabla_\mu\nabla^\mu + m^2 + \xi R)\,G(x, x') = g^{-1/2}(x)\,\delta(x - x') \tag{4}$$

One can solve equation (4) by the method proposed by Schwinger (1951) and DeWitt (1975) and write the solution as

$$G(x, x') = \frac{i}{(4\pi)^2}\,\Delta^{1/2}(x, x') \times \int_0^\infty \frac{i\,ds}{(is)^2}\exp\left(-im^2 s - \frac{\sigma}{2is}\right)\cdot F(x, x'; is) \tag{5}$$

where $\Delta(x, x')$ is the van Vleck determinant defined by the expression

$$\Delta(x, x') = -g^{-1/2}(x) \det[-\partial_\mu \partial_{\nu'} \sigma(x, x')]\, g^{-1/2}(x') \tag{6}$$

and the biscalar $\sigma(x, x')$ is half the square of the geodesic distance between x and x'.

The function $F(x, x'; is)$ has the following adiabatic expression:

$$F(x, x'; is) = f_0(x, x') + f_1(x, x')\, is + f_2(x, x')(is)^2 + \cdots \tag{7}$$

where

$$\begin{aligned} f_0(x, x') &= 1 \\ f_1(x, x') &= (\tfrac{1}{6} - \xi)\, R + \tfrac{1}{2}(\tfrac{1}{6} - \xi)\, R_{;\alpha}\, y^\alpha \\ &\quad - \tfrac{1}{3} a_{\alpha\beta} \cdot (x - x')^\alpha\, (x - x')^\beta \\ f_2(x, x') &= \tfrac{1}{2}(\tfrac{1}{6} - \xi)^2\, R^2 - \tfrac{1}{3} a^\lambda_\lambda \end{aligned} \tag{8}$$

and

$$\begin{aligned} a_{\alpha\beta} = {} & \tfrac{1}{2}(\xi - \tfrac{1}{6})\, R_{;\alpha\beta} + \tfrac{1}{120} R_{;\alpha\beta} - \tfrac{1}{40} \Box R_{\alpha\beta} \\ & + \tfrac{1}{30} R^\lambda_\alpha R_{\lambda\beta} - \tfrac{1}{60} R^{k\lambda}_{\alpha\beta} R_{k\lambda} - \tfrac{1}{60} R^{\lambda\mu k}_\alpha R_{\lambda\mu k\beta} \end{aligned}$$

are the Minakshisundaram coefficients.

We can write also that $G(x, x')$ is given by

$$G(x, x') = \int_0^\infty i\, ds\, \mathscr{G}(x, x'; is) \tag{9}$$

where

$$\mathscr{G}(x, x'; is) = \frac{i\Delta^{1/2}(x, x')}{(4\pi is)^2} \exp\left(-ism^2 - \frac{\sigma}{2is}\right) F(x, x'; is) \tag{10}$$

is the heat kernel for operator K. It is ruled by the equation

$$i \frac{\partial}{\partial s} \mathscr{G}(x, x'; is) = K_{xz} \mathscr{G}(z, x'; is) \tag{11}$$

with the boundary conditions $\mathscr{G}(x, x'; 0) = \delta(x - x')$. One can show that the functional determinant of the operator K is defined as

$$\ln \operatorname{Det} K = -\int_0^\infty i\, ds (is)^{-1} \operatorname{tr} \mathscr{G}(x, x'; is) \tag{12}$$

and, therefore,

$$W = -\frac{i}{2}\int_0^{\infty} i\,ds(is)^{-1}\,tr\,\mathscr{G}(x, x'; is) \tag{13}$$

Inserting (10) into (13) and using (9), we write the effective action in the form

$$W = -\frac{i}{2}\int d^4x\sqrt{g}\int_{m^2}^{\infty} dm^2\,G(x, x) \tag{14}$$

The thermodynamic properties of the Bose gas can be studied after the thermodynamic potential is found,

$$\Omega = -T\ln Z \tag{15}$$

where Z is the partition function. On the other hand, it is known that the thermodynamic potential is equal to the effective Lagrangian of the scalar field at finite temperature, that is,

$$\Omega = -W[\beta] \tag{16}$$

In the next section we show how to get the densities of quantum Bose gas thermodynamic potentials using (15) and (16).

3. LOCAL MOMENTUM-SPACE REPRESENTATION OF GREEN'S FUNCTION

We need to get the expression for the Green's function at finite temperature to find the thermodynamic potential of the Bose gas according to (14) and (16). The expression (14) contains the Green's function in the limit of $x \to x'$. Therefore one can use the method of momentum-space representation for the Green's function (Bunch and Parker, 1979) as the most convenient for our thermodynamic constructions.

The Green's function as the solution of equation (3) can be obtained in a local region of Riemann space-time by introducing Riemann normal coordinates (Petrov, 1969). Let x and x' be points of Riemann space-time. Normal coordinates y with origin at point x determine the position of any point x' near point x as

$$y^{\mu} = \tau \cdot \xi^{\mu}$$

where ξ^{μ} is the unit tangent vector (with the origin at point x) to the geodesic which joins points x and x', and τ is the arc length along this geodesic.

One can write the expressions for the metric tensor in the Riemann normal coordinates as

$$g_{\mu\nu} = \eta_{\mu\nu} - \tfrac{1}{3} R_{\mu\alpha\nu\beta} y^\alpha y^\beta - \tfrac{1}{6} R_{\mu\alpha\nu\beta;\gamma}\, y^\alpha y^\beta y^\gamma \\ + \left(-\tfrac{1}{20} R_{\mu\alpha\nu\beta;\gamma\delta} + \tfrac{2}{45} R_{\alpha\mu\beta\lambda} R^\lambda_{\gamma\nu\delta}\right) y^\alpha y^\beta y^\delta + \cdots \tag{17}$$

where $\eta_{\mu\nu}$ is the Minkowski metric, and

$$g = 1 - \tfrac{1}{3} R_{\alpha\beta}\, y^\alpha y^\beta - \tfrac{1}{6} R_{\alpha\beta;\gamma}\, y^\alpha y^\beta y^\gamma \\ + \left(\tfrac{1}{16} R_{\alpha\beta} R_{\gamma\delta} - \tfrac{1}{90} R^k_{\lambda\alpha\beta} R^\lambda_{\gamma\delta k} - \tfrac{1}{20} R_{\alpha\beta;\gamma\delta}\right) y^\alpha y^\beta y^\gamma y^\delta + \cdots \tag{18}$$

The coefficients of the expressions (17) and (18) are evaluated at $y = 0$. For the solution of equation (4) written in normal coordinates one can use an iterative procedure. The resulting expression for the Green's function may be written in the form

$$G(x, y) = \Delta^{1/2}(x, y) \sum_{j=0}^{\infty} f_j(x, y) \left(-\frac{\partial}{\partial m^2}\right)^j G_0(y) \tag{19}$$

where

$$G_0(y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{iky}}{k^2 + m^2} \\ ky = k_\alpha\, y^\alpha = \eta^{\alpha\beta} k_\alpha\, y_\beta \tag{20}$$

In the limit of $x = x'$, according to (8) the functions □ are given by

$$f_0(x) = 1, \qquad f_1(x) = \left(\tfrac{1}{6} - \xi\right) R \\ f_2(x) = \tfrac{1}{2}\left(\tfrac{1}{6} - \xi\right)^2 R^2 - \tfrac{1}{3} a^\lambda_\lambda \tag{21}$$

4. FINITE-TEMPERATURE GREEN'S FUNCTION. DENSITY OF THERMODYNAMIC POTENTIAL

The expression (19) enables us to introduce the finite temperature in the same way as in Euclidean quantum field theory. The normal route to introducing finite temperature is the following. Let the coordinate y^0 of Euclidean space-time change in a segment $y^0 \in [0, -i\beta]$, where $\beta^{-1} = T$ is the temperature. Then the finite-temperature Green's function can be found by the replacement

$$\int \frac{d^4k}{(2\pi)^4} f(k) \to \frac{i}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} f(\omega_n, \mathbf{k}) \tag{22}$$

where

$$k^0 \to \omega_n = 2\pi n i T, \qquad n = 0, \pm 1, \pm 2, \ldots$$
$$k^2 = -\omega_n^2 + \mathbf{k}^2$$

Therefore one can write equation (20) in the form

$$G_0 = \frac{i}{\beta} \int \frac{d^3k}{(2\pi)^3} \sum_{n=-\infty}^{\infty} \frac{1}{-\omega_n^2 + \varepsilon^2} \tag{23}$$

where $\varepsilon^2 = (\mathbf{k}^2 + m^2)^{1/2}$ is the energy of the particle.

Making the summation

$$\frac{1}{\beta} \sum_{n=-\infty}^{\infty} \frac{1}{-\omega_n^2 + \varepsilon^2} = \frac{1}{2\varepsilon} + \frac{1}{\varepsilon(e^{\beta\varepsilon} - 1)} \tag{24}$$

we obtain in the limit of $x \to x'$

$$G_{T \neq 0}(x, x) = G(x, x) + G_\beta(x) \tag{25}$$

The finite-temperature Green's function of the scalar field is the sum of the Green's function $G(x, x)$ and the finite-temperature part in the form

$$G_\beta(x) = i \sum_{j=0}^{\infty} f_j(x) \left(-\frac{\partial}{\partial m^2} \right)^j \int \frac{d^3k}{(2\pi)^3} \frac{1}{\varepsilon(e^{\beta\varepsilon} - 1)} \tag{26}$$

Using equations (14), (16), and the results of this section, we find that the density of the thermodynamic potential may be written as

$$\Omega(x) = \tfrac{1}{2} \int_{m^2}^{\infty} dm^2 \, G_\beta(x) \tag{27}$$

5. STATISTICS OF BOSE GAS. BOLTZMANN GAS

The infinite contribution of the zero-temperature Green's function $G(x, x)$, (25), to the value of the thermodynamic potential $\Omega(x)$, (27), can be eliminated by means of a renormalization procedure in the full Lagrangian which includes a gravitational part and a material part. We discuss this problem in Section 7.

Inserting (26) into (27), we get

$$\Omega(x) = -\frac{1}{2} \sum_{j=0}^{\infty} f_j(x) \int_{m^2}^{\infty} dm^2 \left(-\frac{\partial}{\partial m^2} \right)^j \times \left[\int \frac{d^3k}{(2\pi)^3} \frac{1}{\varepsilon(e^{\beta\varepsilon} - 1)} \right] \tag{28}$$

Since

$$-\frac{2}{\beta}\frac{\partial}{\partial m^2}\ln(1-e^{-\beta\varepsilon})=\frac{1}{\varepsilon(e^{\beta\varepsilon}-1)}$$

we find that

$$\Omega(x)=\frac{1}{\beta}\sum_{j=0}^{\infty} f_j(x)\left(-\frac{\partial}{\partial m^2}\right)^j\int\frac{d^3k}{(2\pi)^3}\ln(1-e^{-\beta\varepsilon}) \tag{29}$$

To determine this equation at zero momentum we include the chemical potential μ and fugacity $z=\exp(\beta\mu)$. The chemical potential can be introduced as $(k^0)^2=\omega_n^2+\mu$ (Kapusta, 1979). Then

$$\Omega(x)=\frac{1}{\beta}\sum_{j=0}^{\infty} f_j(x)\left(-\frac{\partial}{\partial m^2}\right)^j\int\frac{d^3k}{(2\pi)^3}\ln(1-ze^{-\beta\varepsilon}) \tag{30}$$

Introducing the function

$$g_{5/2}(z)=-\frac{4}{\sqrt{\pi}}\int dx\, x^2\ln(1-ze^{-x^2})=\sum_{l=1}^{\infty}\frac{z^l}{l^{5/2}} \tag{31}$$

one can write the expression for the thermodynamic potential in curved space-time as

$$\Omega(x)=\sum_{j=0}^{\infty} f_j(x)\, g_{5/2}(x)\left(-\frac{\partial}{\partial m^2}\right)^j\frac{1}{\lambda^3} \tag{32}$$

where $\lambda=(2\pi/mT)^{1/2}$ is the thermal wavelength of the particle.

The average number of particles in a certain momentum state is obtained as the derivative

$$\langle n_{\mathrm{k}}\rangle=-\frac{\partial\Omega(x)}{\partial\mu}=\sum_{j=0}^{\infty} f_j(x)\left(-\frac{\partial}{\partial m^2}\right)^j\frac{1}{z^{-1}e^{\beta\varepsilon}-1} \tag{33}$$

The dependence of the fugacity z on the curvature of the manifold, the temperature, and the particle density is given by the expression

$$n=\frac{1}{\lambda^3}\left[1-f_1(x)\frac{3}{4m^2}-f_2(x)\frac{3}{16m^4}+\cdots\right]g_{3/2}(z)+\frac{z}{1-z} \tag{34}$$

where

$$g_{3/2}(z)=z\frac{\partial}{\partial z}g_{5/2}(z) \tag{35}$$

Therefore the chemical potential, density of particles, and temperature are connected with the curvature of space-time.

Now one can consider the Boltzmann distribution as the high-temperature limit of the Bose distribution at small particle density. Let $\lambda^3 n \ll 1$; then, in the approximation of the second order in curvature we can write

$$\langle n_{\mathbf{k}} \rangle = z \cdot \left[1 + f_1(x) \left(-\frac{\partial}{\partial m^2} \right) + \cdots \right] e^{-\beta \varepsilon_{\mathbf{k}}} \tag{36}$$

So the gravitational field changes the value of the chemical potential as

$$\mu(T, R) \approx \mu_0(T) - \frac{1}{2m} \left(\frac{1}{6} - \xi \right) R \tag{37}$$

The formula (37) determines the dependence of the chemical potential of the Boltzmann gas on curvature R.

6. BOSE–EINSTEIN CONDENSATION

Another example of the possibilities of the proposed scheme is the phenomenon of Bose–Einstein condensation. This phenomenon was investigated by Altaie (1978) and Singh and Pathria (1984) in the space-time of constant positive curvature. But we shall define the dependence of the critical temperature on the geometry of the space-time manifold according to our method.

At the critical temperature of Bose–Einstein condensation $z = 1$. Then the formula (34) leads to

$$n\lambda^3 \geqslant \zeta \left(\frac{3}{2} \right) \left[1 - f_1(x) \frac{3}{4m^2} - \cdots \right] \tag{38}$$

where $\zeta(3/2)$ is the Riemann zeta function.

Unfortunately, (38) is very nonlinear, but in the simple case of the approximation of the gravitational field the critical temperature of the condensation is

$$T_c(R) = T_c \left[1 + \frac{1}{2} \frac{f_1(x)}{m^2} + \cdots \right] \tag{39}$$

where

$$T_c = T_c(R = 0) = \frac{2\pi}{m} \left[\frac{n}{\zeta(3/2)} \right]^{2/3}$$

The behavior of the chemical potential in curved space-time is shown in Fig. 1.

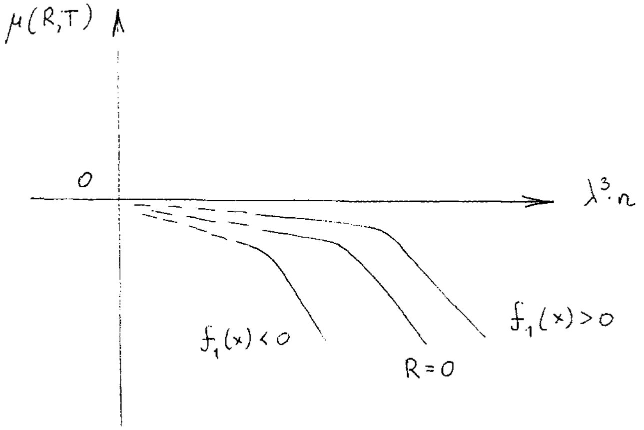

Fig. 1. The dependence of the chemical potential on the curvature.

7. DIVERGENCES IN ONE-LOOP EFFECTIVE ACTION AT FINITE TEMPERATURE

As one can show by calculations, the contribution $G(x, x)$ in the expression (25) is divergent (Birrell and Davies, 1982). To have a clear picture of the model under consideration we need to eliminate these divergent contributions.

Let the complete Lagrangian be

$$\mathscr{L} = \mathscr{L}_g + \mathscr{L}_m \tag{40}$$

where gravitational Lagrangian is

$$\mathscr{L}_g = \frac{1}{16\pi G_0}(R - 2\Lambda_0) + \alpha_0 R^2 + \beta_0 R_{\alpha\beta} R^{\alpha\beta} + \gamma_0 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \tag{41}$$

and $\mathscr{L}_m$ coincides with (1).

The finite-temperature one-loop effective action is

$$W_g + W_m \xrightarrow{T \neq 0} W_g + W_m[0] + W_m[\beta] \tag{42}$$

where

$$W_m[0] = \int d^4x \sqrt{g}\, \mathscr{L}_{\text{eff}}(x) \tag{43}$$

and $W_n[\beta]$ leads to the thermodynamic potential.

One can find the expression for $\mathscr{L}_{\text{eff}}(x)$ using the results of Section 2. Inserting (5) into (14) and making the integration, we get

$$\mathscr{L}_{\text{eff}}(x)=\frac{1}{2(4\pi)^2}\sum_{j=0}^{\infty} f_j(x)\int_0^{\infty} i\,ds(is)^{j-3}\,e^{-im^2s} \tag{44}$$

Let us use the procedure of dimensional regularization to select divergent terms. The expression (44) can be rewritten in n-dimensional space-time in the following way:

$$\mathscr{L}_{\text{eff}}(x)=\frac{1}{2}(4\pi)^{-n/2}\left(\frac{m}{M}\right)^{n-4}\sum_{j=0}^{\infty} f_j(x)\,m^{4-2j}\Gamma\left(j-\frac{n}{2}\right) \tag{45}$$

where M is an arbitrary mass scale.

The first three terms are divergent because the Γ function has poles at $n\to 4$. Therefore the divergent contributions in $\mathscr{L}_{\text{eff}}(x)$ are

$$\begin{aligned}\mathscr{L}_{\text{div}} = -(4\pi)^{-n/2}&\left\{\frac{1}{n-4}+\frac{1}{2}\left[j+\ln\left(\frac{m^2}{M^2}\right)\right]\right\}\\ &\times\left[\frac{4m^2}{n(n-2)}-\frac{2}{n-2}m^2 f_1(x)+f_2(x)\right]\end{aligned} \tag{46}$$

Combining (46) together with (41), we can redetermine the constants of the gravitation Lagrangian density as

$$\frac{1}{8\pi G_R}\Lambda_R=\frac{1}{8\pi G_0}\Lambda_0-\frac{1}{(4\pi)^2}\frac{1}{n-4}\frac{m^2}{2}$$

$$\frac{1}{16\pi G_R}=\frac{1}{16\pi G_0}+\frac{1}{(4\pi)^2}\frac{1}{n-4}m^2\left(\frac{1}{6}-\xi\right)$$

$$\alpha_R=\alpha_0-\frac{1}{(4\pi)^2}\frac{1}{n-4}\frac{1}{2}\left(\frac{1}{6}-\xi\right)^2$$

$$\beta_R=\beta_0+\frac{1}{180}\frac{1}{(4\pi)^2}\frac{1}{n-4}$$

$$\gamma_R=\gamma_0-\frac{1}{180}\frac{1}{(4\pi)^2}\frac{1}{n-4}$$

where G_0, Λ_0, α_0, β_0, and γ_0 are bare constants and G_R, Λ_R, α_R, β_R, and γ_R are the physical (finite) constants.

Since all divergences of $\mathscr{L}_{\text{div}}$ can be included into $\mathscr{L}_g$, only the finite-temperature contributions observed in Section 3 remain.

8. CONCLUSION

The method developed in this paper bears a relation to the problems of quantum statistics and local thermodynamics of an ideal Bose gas in curved space-time and is based on the finite-temperature Green's function method. Unlike the ideas suggested by Dowker (1978) and Semenoff and Weiss (1985), we believe that it is enough to know the Green's functions only in the local sphere of the manifold. This can be done by using the local momentum-space representation of the Green's function in external curved space-time. In our model the chemical potential and occupation numbers depend on the curvature of the manifold in which the system is considered. The critical temperature of Bose–Einstein condensation is a function of curvature, too. A similar conclusion was reached in the works of Altaie (1978) and Singh and Pathria (1984) in simpler cases. As all thermodynamic values are functions of a point of space-time, this means that our system is in a nonequilibrium state. Since the point is chosen in an arbitrary way, we believe that the proposed method gives the possibility to describe the quantum statistics and thermodynamics of nonstationary processes because the result of measurements depends on the position of an observer only. Such a point of view was first expressed by Gyarmati (1970), who indicated that the best way to investigate nonstationary processes is to divide the whole volume into spheres of equilibrium. In our opinion this is achieved in a natural manner by introducing the normal coordinates in finite-temperature field theory in external curved space-time.

The method under consideration can be applied to the investigation of the thermodynamic quantities of a quantum Fermi gas and a photon gas (Pronin and Kulikov, 1987).

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